# ME451: Control Systems

**Lecture 20**

**Root locus: Lead compensator design**

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# Course roadmap

*Modeling*

- ✓ Laplace transform
- ✓ Transfer function
- ✓ Models for systems
  - ✓ electrical
  - ✓ mechanical
  - ✓ electromechanical
- ✓ Block diagrams
- ✓ Linearization

*Analysis*

- ✓ Time response
  - ✓ Transient
  - ✓ Steady state
- Frequency response
  - • Bode plot
- ✓ Stability
  - ✓ Routh-Hurwitz
  - • Nyquist

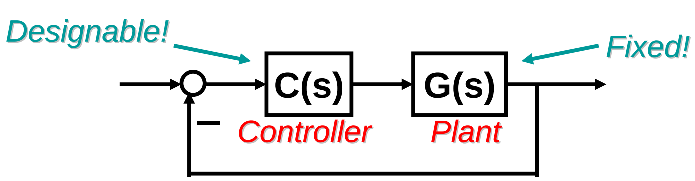
*Design*

- Design specs
- Root locus
- Frequency domain
- PID & Lead-lag
- Design examples

*(Matlab simulations &) laboratories*



# Closed-loop design by root locus



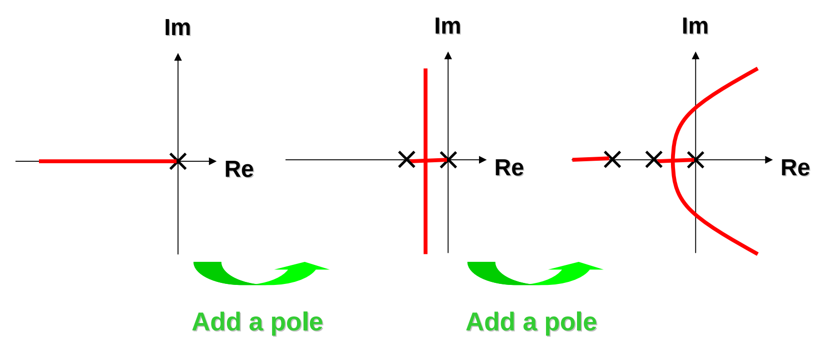

- Place closed-loop poles at desired location
  - by tuning the gain C(s)=K. (for time domain specs)
- If root locus does not pass the desired location, then reshape the root locus
  - by adding poles/zeros to C(s). (How?)

*Compensation*



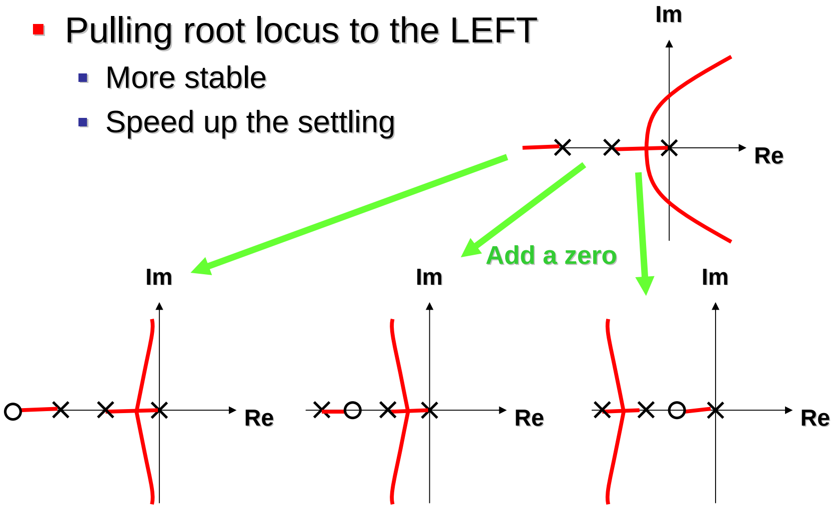
# General effect of addition of poles

- Pulling root locus to the RIGHT
  - Less stable
  - Slow down the settling



# General effect of addition of zeros

- Pulling root locus to the LEFT
  - More stable
  - Speed up the settling



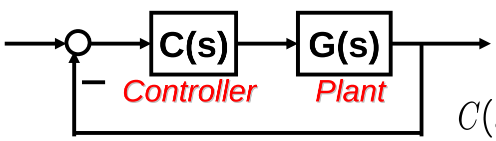



# Some remarks

- Adding only zero $C(s) = s + z, \ (z > 0)$
  - often problematic because such controller amplifies the high-frequency noise.
- Adding only pole $C(s) = 1/(s + p), \ (p > 0)$
  - often problematic because such controller generates a less stable system (by moving the closed-loop poles to the right).
- These facts can be explained by using *frequency response analysis*.
- Add both zero and pole!

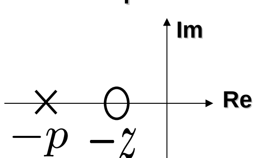


# Lead and lag compensators

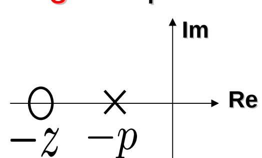



$$C(s) = K\frac{s+z}{s+p}, \ (z > 0, p > 0)$$

- Lead compensator




- Lag compensator



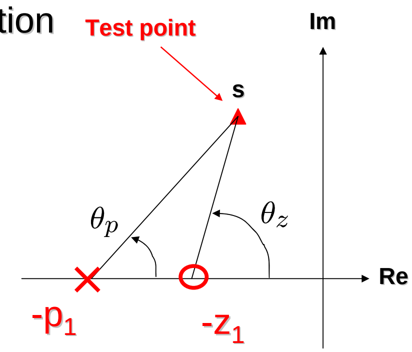

**Why these are called "lead" and "lag"?**
**We will see that from frequency response in this class.**



# Lead compensator

- Positive angle contribution

$$\angle C_{Lead}(s) = \theta_{Lead} > 0$$

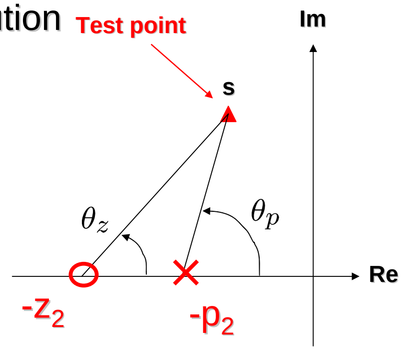



$$\angle C_{Lead}(s) = \angle \frac{s+z_1}{s+p_1} = \angle(s+z_1) - \angle(s+p_1)$$
$$= \theta_z - \theta_p = \theta_{Lead} > 0$$

# Lag compensator

- Negative angle contribution

$$\angle C_{Lag}(s) = \theta_{Lag} < 0$$




$$\angle C_{Lag}(s) = \angle \frac{s+z_2}{s+p_2} = \angle(s+z_2) - \angle(s+p_2)$$
$$= \theta_z - \theta_p = \theta_{Lag} < 0$$



# Roles of lead and lag compensators

- Lead compensator (Today)
  - Improve transient response
  - Improve stability

$$C_{Lead}(s) = K_1 \frac{s+z_1}{s+p_1}$$

- Lag compensator (Next)
  - Reduce steady state error

$$C_{Lag}(s) = K_2 \frac{s+z_2}{s+p_2}$$

- Lead-lag compensator (Next)
  - Take into account all the above issues.

$$C_{LL}(s) = C_{Lead}(s) C_{Lag}(s)$$



# Radar tracking system

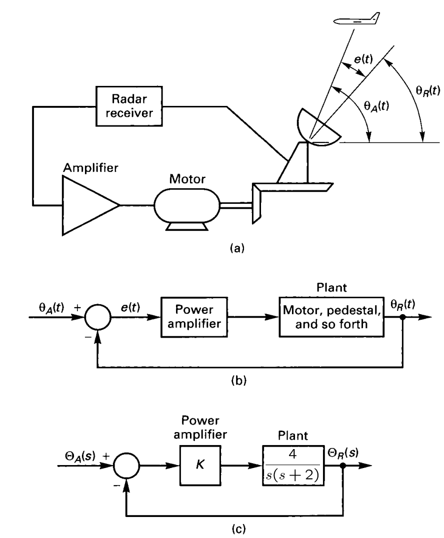


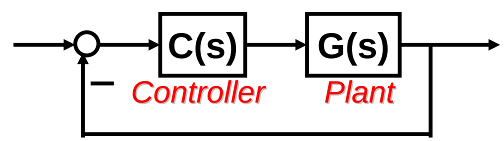

**Figure 7.1** Radar tracking system.



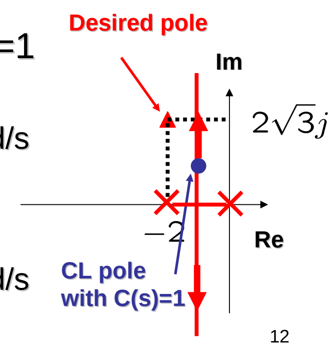

# Lead compensator design

- Consider a system

$$G(s) = \frac{4}{s(s+2)}$$




- Analysis of CL system for C(s)=1
  - Damping ratio ζ=0.5
  - Undamped natural freq. ωn=2 rad/s
- Performance specification
  - Damping ratio ζ=0.5
  - Undamped natural freq. ωn=4 rad/s



# Angle and magnitude conditions (review)

- A point s to be on root locus ←→ it satisfies
  - **Angle condition**

*Odd number*

$$\angle L(s) = 180^o \times (2k+1), \; k = 0, \pm 1, \pm 2, \ldots$$

- For a point on root locus, gain K is obtained by
  - **Magnitude condition**

$$|L(s)| = \frac{1}{K}$$



# Lead compensator design (cont'd)

Evaluate G(s) at the desired pole.

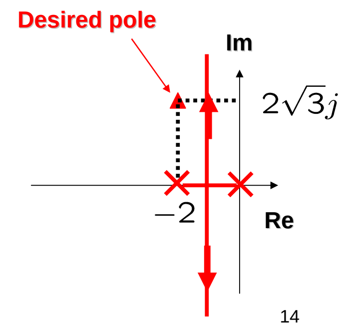
$$G(-2+2\sqrt{3}j) = \frac{4}{(-2+2\sqrt{3}j)2\sqrt{3}j} = \frac{-1}{3+\sqrt{3}j}$$

- o If *angle condition* is satisfied, compute the corresponding K.
- o In this example,

$$\angle G(-2+2\sqrt{3}j) = -210$$

Angle condition is not satisfied.

➡ **Angle deficiency** $\phi = 30$






# Lead compensator design (cont'd)

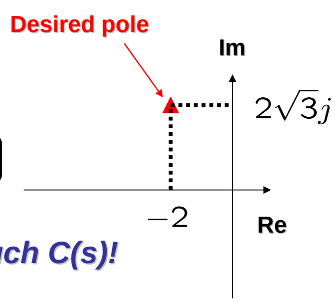
To compensate angle deficiency,
design a lead compensator C(s)

$$C(s) = K\frac{s+z}{s+p}$$

satisfying

$$\angle C(-2+2\sqrt{3}j) = 30 (=: \phi)$$

$$\left( \Rightarrow \angle GC(-2+2\sqrt{3}j) = -180 \right)$$



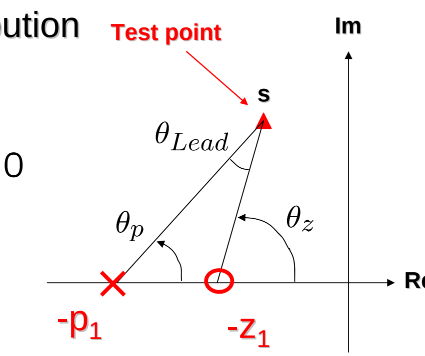

***There are many ways to design such C(s)!***



# Lead compensator

- Positive angle contribution

$$\angle C_{Lead}(s) = \theta_{Lead} > 0$$




- Triangle

$$\theta_p + \theta_{Lead} + (\pi - \theta_z) = \pi$$

$$\theta_z - \theta_p = \theta_{Lead}$$

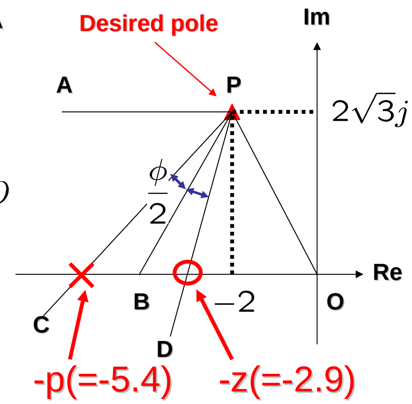
# How to select pole and zero?

- Draw horizontal line PA
- Draw line PO
- Draw bisector PB

$$\angle APB = \angle BPO = \frac{1}{2}\angle APO$$

- Draw PC and PD

$$\angle CPB = \angle BPD = \frac{\phi}{2}$$

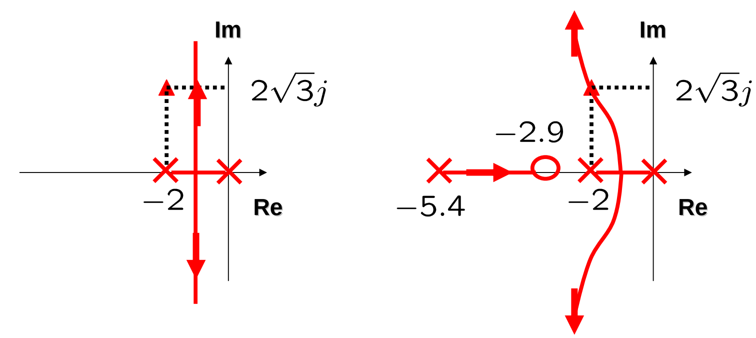



- Pole and zero of C(s) are shown in the figure.



# Comparison of root locus

- G(s)
- G(s)C(s)




**Improved stability!**



# How to design the gain K?

- Lead compensator

$$C(s) = K\frac{s+2.9}{s+5.4}$$

- Open loop transfer function

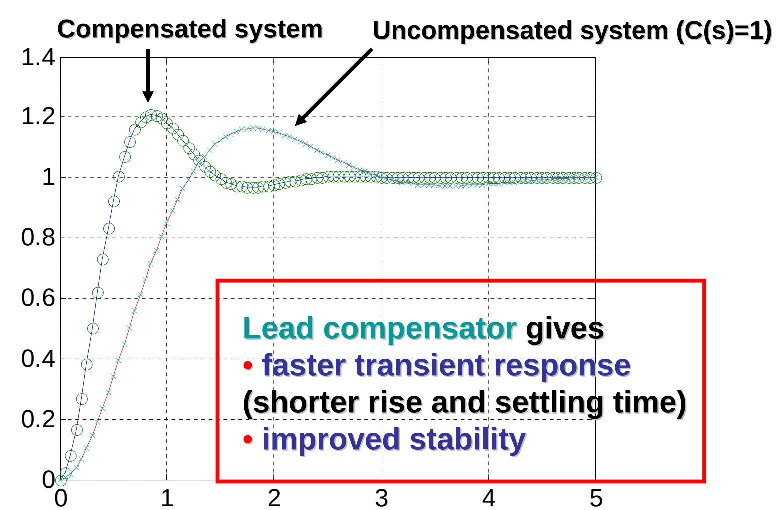
$$G(s)C(s) = K\frac{4(s+2.9)}{s(s+2)(s+5.4)}$$

- Magnitude condition

$$K\left|\frac{4(s+2.9)}{s(s+2)(s+5.4)}\right|_{s=-2+2\sqrt{3}j} = 1 \Rightarrow K = 4.675$$



# Comparison of step responses



# Error constants

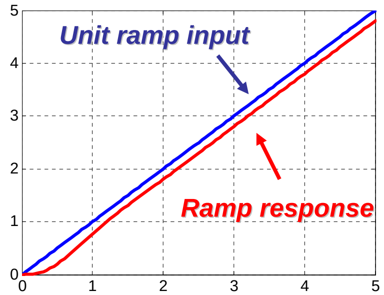
$$G(s)C(s) = \frac{4}{s(s+2)} \cdot \frac{4.675(s+2.9)}{s+5.4}$$

- Step-error constant

$$K_p := \lim_{s \to 0} G(s)C(s) = \infty$$

- Ramp-error constant

$$K_v := \lim_{s \to 0} sG(s)C(s) = 5.02$$




*Lag compensator can be used to reduce steady-state error. (Next lecture)*



# Summary and exercises

- Controller design based on root locus
  - General effects of addition of pole and zero
  - Lead lag compensator realization with op amp
  - Lead compensator design
    - **Lead compensator improves stability and transient response.**
- Next, lag & lead-lag compensator design