ME451: Control Systems

Relative stability

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Gain margin (GM)

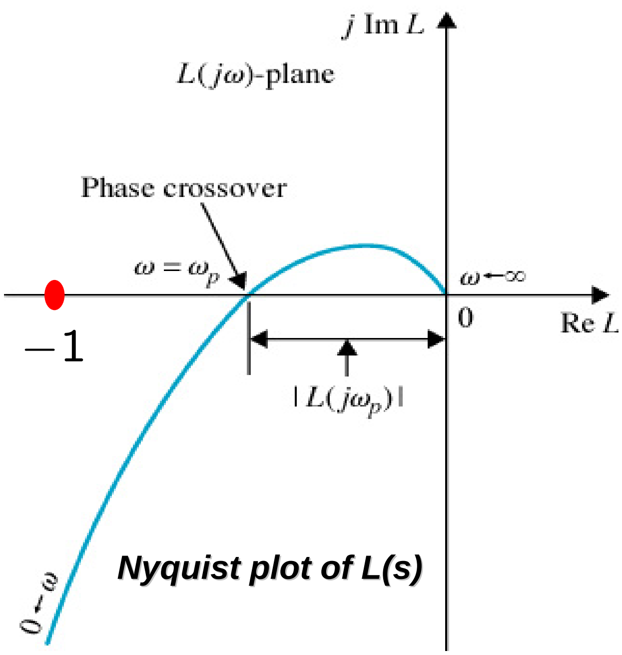
- Phase crossover frequency ω_p:

$$\angle L(j\omega_p) = -180$$

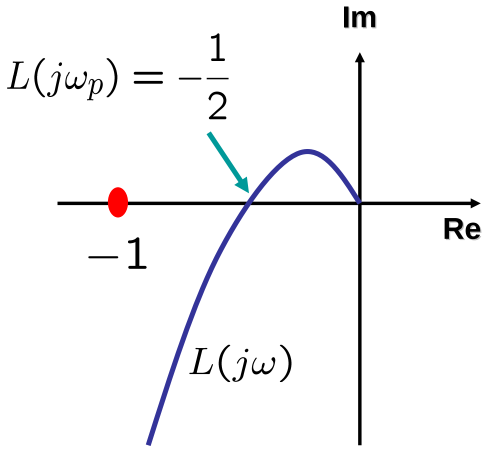
- Gain margin (in dB)

$$GM = 20\log_{10}\frac{1}{|L(j\omega_p)|}$$

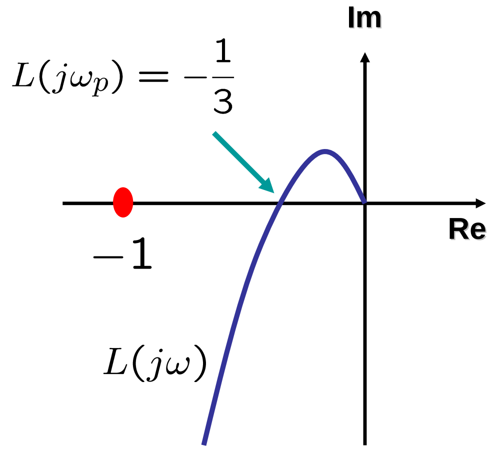
- Indicates how much OL gain can be multiplied without violating CL stability.



Examples of GM

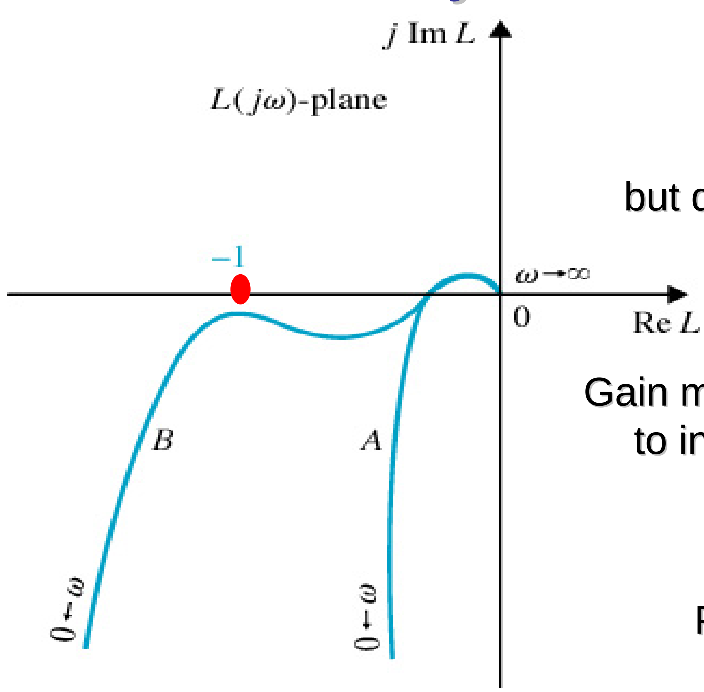


$$GM = 20\log_{10}\underbrace{\frac{1}{|L(j\omega_p)|}}_{2} \approx 6\text{dB}$$



$$GM = 20\log_{10}\underbrace{\frac{1}{|L(j\omega_p)|}}_{3} \approx 9.5\text{dB}$$

Reason why GM is inadequate



Same gain margin,
but different relative stability

Gain margin is often inadequate
to indicate relative stability

Phase margin!

Phase margin (PM)

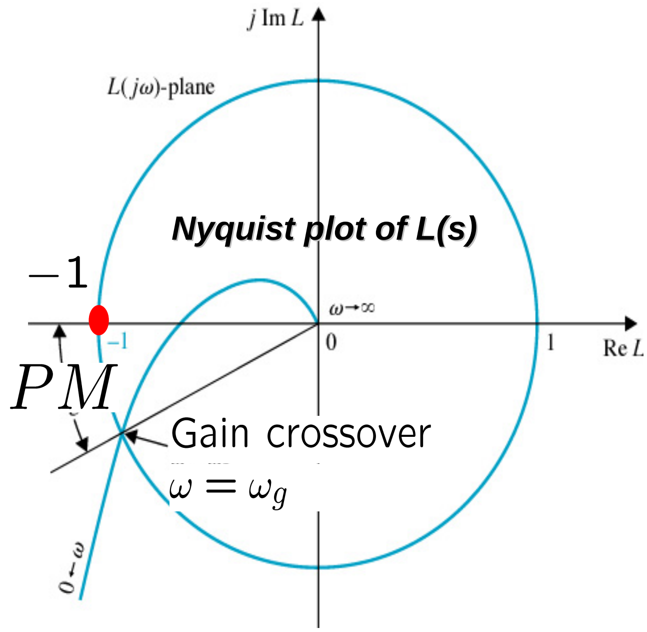
- Gain crossover frequency ω_g:

$$|L(j\omega_g)| = 1$$

- Phase margin

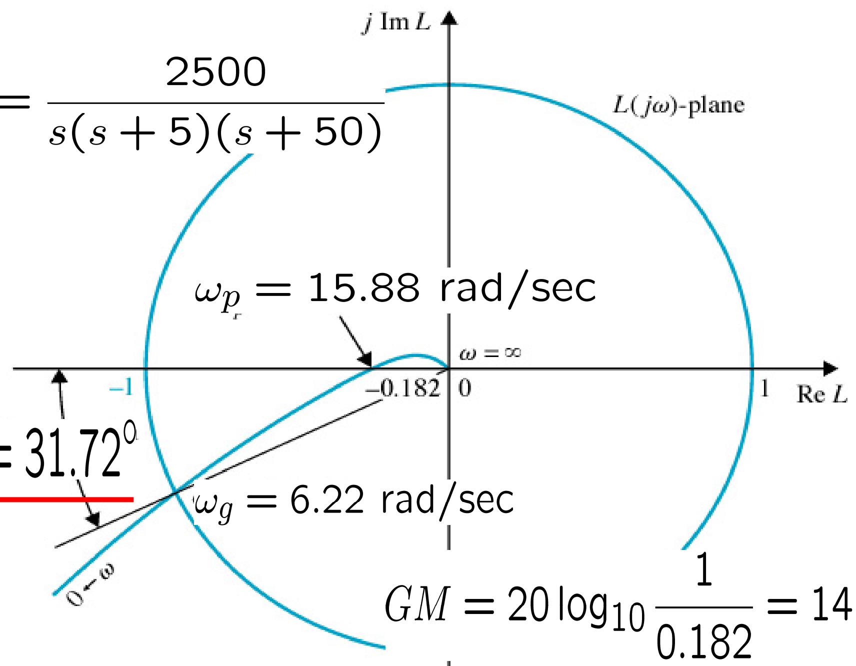
$$PM = \angle L(j\omega_g) - 180^\circ$$

- Indicates how much OL phase can be added without violating CL stability.



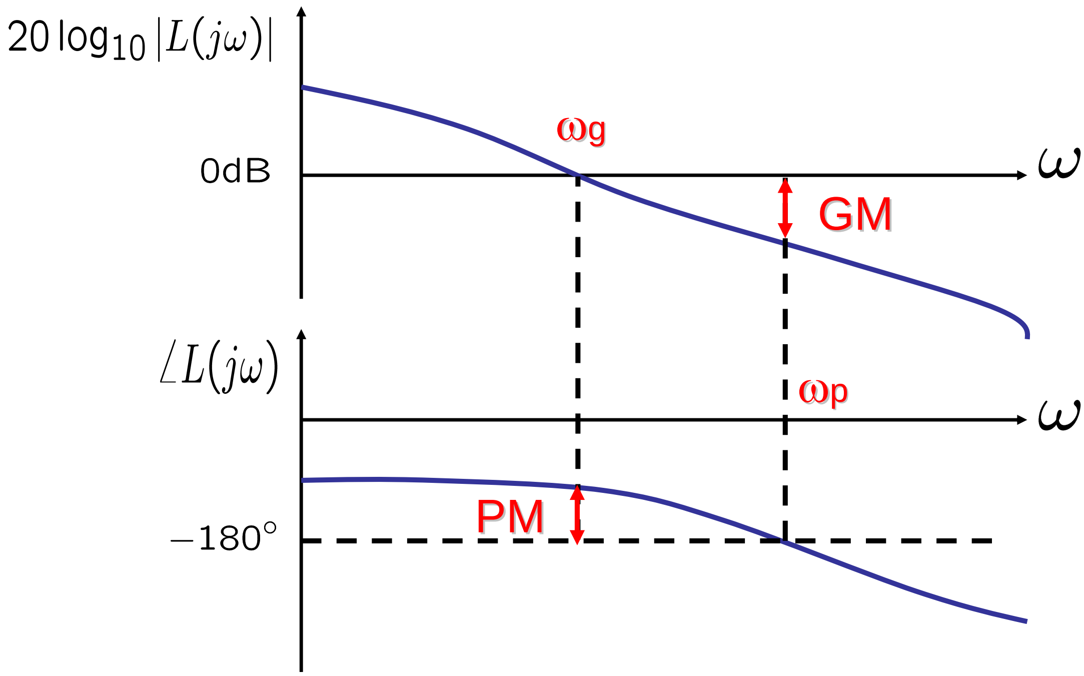
Example 9.5

$$L(s) = \frac{2500}{s(s+5)(s+50)}$$

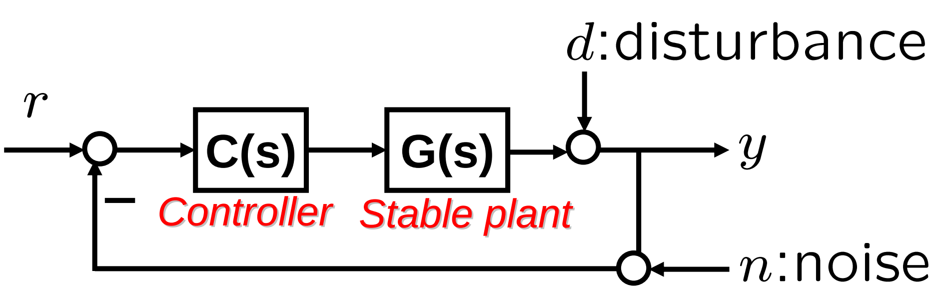


$$GM = 20\log_{10}\frac{1}{0.182} = 14.80\text{dB}$$

Relative stability on Bode plot



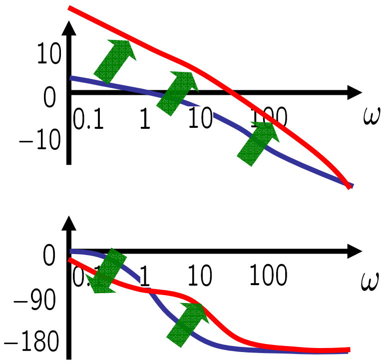
Frequency shaping (Loop shaping)



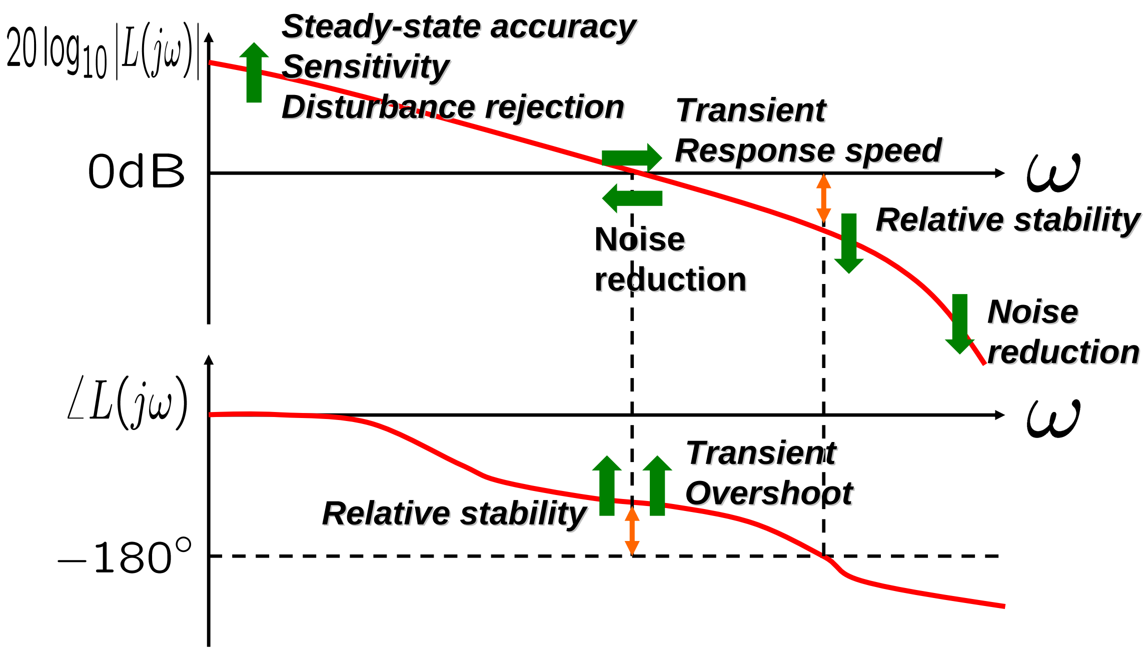
Reshape Bode plot of G(jω) into a "desired" shape of

$$L(j\omega) := G(j\omega)C(j\omega)$$

by a series connection of appropriate C(s).



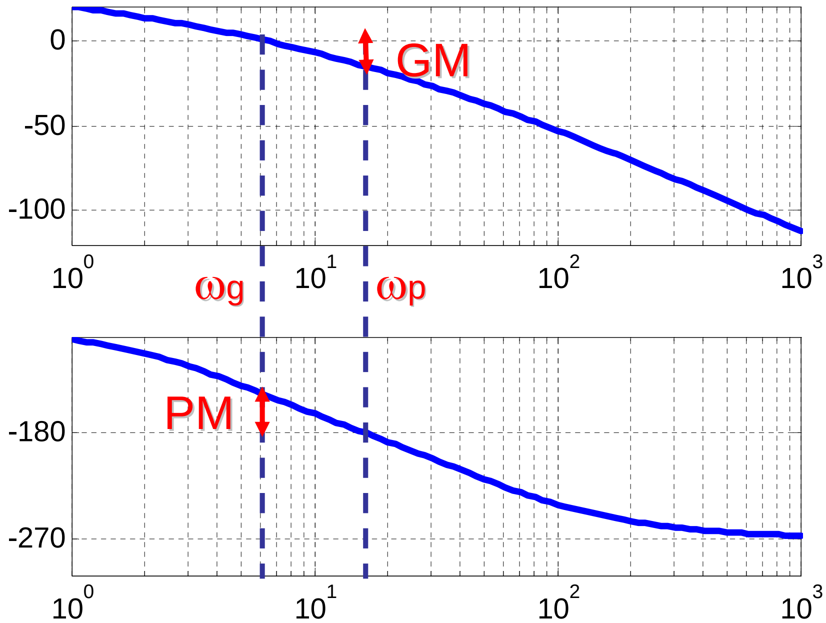
Typical shaping goal (review)



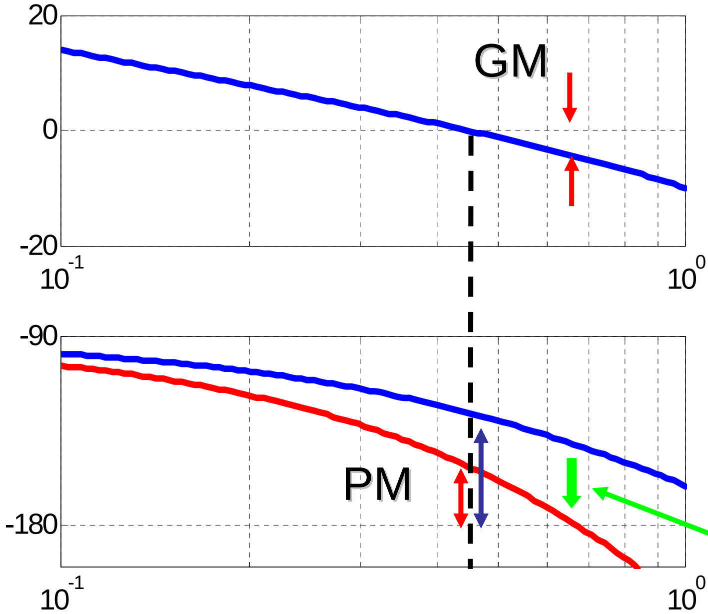
Notes on Bode plot

- Advantages
  - Without computer, Bode plot can be sketched easily.
  - GM, PM, crossover frequencies are easily determined on Bode plot.
  - Controller design on Bode plot is simple. (Next week)
- Disadvantage
  - If OL system is unstable, we cannot use Bode plot for stability analysis.

Example 9.6



Relative stability with time delay



$$L(s) = \frac{1}{s(s+1)(s+2)}$$

$$L(s) = \frac{e^{-s}}{s(s+1)(s+2)}$$

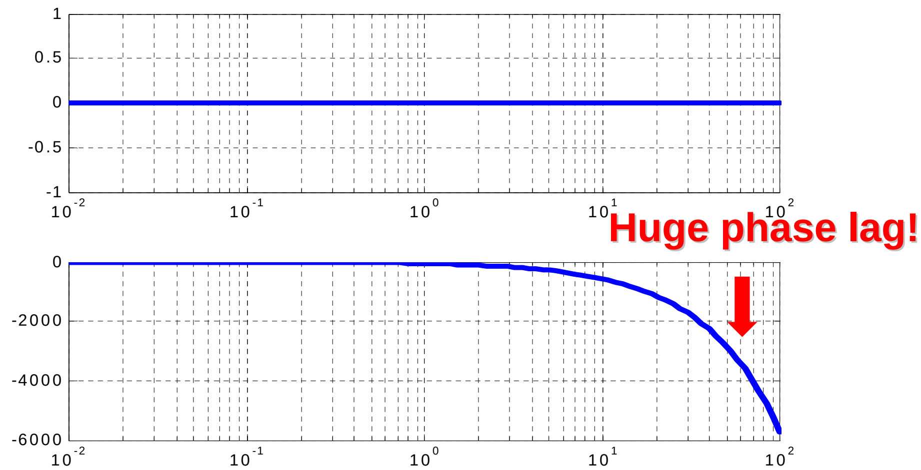
Time delay reduces relative stability!

$57.3\omega T_d$ deg phase lag

Delay time

Bode plot of a time delay (review)

$$G(s) = e^{-Ts} \Rightarrow |G(j\omega)| = 1, \forall\omega,\ \angle G(j\omega) = -\omega T (\text{rad})$$



The phase lag causes instability of the closed-loop system, and thus, the difficulty in control.