Relative stability

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Gain margin (GM)

- Phase crossover frequency $\omega_p$:
  \[ \angle L(j\omega_p) = -180 \]
- Gain margin (in dB)
  \[ GM = 20 \log_{10} \frac{1}{|L(j\omega_p)|} \]
- Indicates how much OL gain can be multiplied without violating CL stability.

Nyquist plot of $L(s)$
Examples of GM

\[ L(j\omega_p) = \frac{1}{2} \]

\[ L(j\omega) \quad \text{Im} \]
\[ \text{Re} \]

\[ L(j\omega) \quad \text{Im} \]
\[ \text{Re} \]

\[ GM = 20 \log_{10} \left( \frac{1}{|L(j\omega_p)|} \right) \approx 6\text{dB} \]

\[ GM = 20 \log_{10} \left( \frac{1}{|L(j\omega_p)|} \right) \approx 9.5\text{dB} \]

Reason why GM is inadequate

Same gain margin, but different relative stability

Gain margin is often inadequate to indicate relative stability

Phase margin!
Phase margin (PM)

- Gain crossover frequency $\omega_g$:
  $$|L(j\omega_g)| = 1$$
- Phase margin
  $$PM = \angle L(j\omega_g) - 180^\circ$$
- Indicates how much OL phase can be added without violating CL stability.

Example 9.5

$$L(s) = \frac{2500}{s(s + 5)(s + 50)}$$

- Gain crossover frequency $\omega_g = 6.22$ rad/sec
- Phase margin $PM = 31.72^\circ$
- Gain margin $GM = 20 \log_{10} \frac{1}{0.182} = 14.80$ dB
Relative stability on Bode plot

Frequency shaping (Loop shaping)

Reshape Bode plot of $G(j\omega)$ into a “desired” shape of $L(j\omega) := G(j\omega)C(j\omega)$ by a series connection of appropriate $C(s)$. 
Typical shaping goal (review)

- Steady-state accuracy
- Sensitivity
- Disturbance rejection
- Transient response speed
- Relative stability
- Noise reduction
- Transient response speed
- Relative stability
- Noise reduction

Notes on Bode plot

- **Advantages**
  - Without computer, Bode plot can be sketched easily.
  - GM, PM, crossover frequencies are easily determined on Bode plot.
  - Controller design on Bode plot is simple. (Next week)

- **Disadvantage**
  - If OL system is unstable, we cannot use Bode plot for stability analysis.
Example 9.6

Relative stability with time delay

\[ L(s) = \frac{1}{s(s + 1)(s + 2)} \]

Time delay reduces relative stability!

57.3\(\omega T_d\) deg phase lag

Delay time
Bode plot of a time delay (review)

\[ G(s) = e^{-Ts} \Rightarrow |G(j\omega)| = 1, \forall \omega, \angle G(j\omega) = -\omega T (\text{rad}) \]

The phase lag causes instability of the closed-loop system, and thus, the difficulty in control.

Huge phase lag!